

An Indirect Adaptive Control Scheme in the Presence of Actuator and Sensor Failures

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The problem of controlling a system in the presence of unknown actuator and sensor faults is addressed. The system is assumed to have groups of actuators, and groups of sensors, with each group consisting of multiple redundant similar actuators or sensors. The types of actuator faults considered consist of unknown actuators stuck in unknown positions, as well as reduced actuator effectiveness. The sensor faults considered include unknown biases and outages. The approach employed for fault detection and estimation consists of a bank of Kalman filters based on multiple models, and subsequent control reconfiguration to mitigate the effect of biases caused by failed components as well as to obtain stability and satisfactory performance using the remaining actuators and sensors. Conditions for fault identifiability are presented, and the adaptive scheme is applied to an aircraft flight control example in the presence of actuator failures. Simulation results demonstrate that the method can rapidly and accurately detect faults and estimate the fault values, thus enabling safe operation and acceptable performance in spite of failures.

I. Introduction

COMPONENT failures in dynamic systems can cause loss of performance and even catastrophic instability. In particular, actuator and sensor failures in flight control systems can result in loss of control leading to serious incidents. Therefore, fault-tolerant control (FTC) has been an active area of research for the past several years. The research in this area has essentially progressed in two directions: an *indirect* adaptive control approach consisting of parameter estimation and **fault detection, isolation/identification**, and control **reconfiguration** (FDIR); and *direct* adaptive control [1-5], wherein the control law is directly updated using the difference between the actual and desired response, without explicit parameter estimation and FDIR. A considerable amount of research has focused on both approaches. Some of the literature on indirect adaptive (reconfigurable) control deals with faults, e.g., [6, 7]; however, adequate attention has not been paid to design of fault detection and isolation/identification (FDI) and FTC in an integrated manner for online and real-time applications using indirect adaptive methods.

The growing demand for reliability and safety in dynamic systems has resulted in significant stimulation of research in fault-tolerant control. A recent survey paper [8] provides bibliographical review of FTC. In general, existing fault-tolerant control system design methods are based on the following major approaches: linear quadratic regulator (LQR)[9], eigenstructure assignment [10], multiple models [11], pseudoinverse [12], neural networks [13], and adaptive control[1-5]. Some FTC methods include a strategy involving a fast subsystem for FDI and a supervisory system that chooses the corresponding controller for a particular type of fault. The most researched and

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mature area in FDI is the residual generation approach using observers [14]. Recently, several new FDI methods, such as Extended Kalman filter (EKF) and Variable Structure Filter, agent-based diagnosis, and unknown input observers based method, etc, have been developed for fault detection and identification [15-19].

In much of the literature, FDI and FTC are studied independently. More specifically, most of the FDI techniques are developed as a diagnostic or monitoring tool, rather than an integral part in FTC. One notable exception is multiple-model (MM) based methods that employ banks of Kalman filters. MM-based adaptive control has resulted in different approaches: the multiple model adaptive control (MMAC) [20], multiple model adaptive estimation (MMAE)-based control [21, 22], and multiple model switching and tuning (MMST) [23, 24]. These methods employ banks of observers or Kalman filters (KF) wherein each observer or KF is tuned to one of several models corresponding to a set of parameters or a failure scenario. The MMAC method and the MMAE method employ KFs and probability-based hypothesis testing to determine the correct model. The MMAC method was originally developed to accommodate varying flight conditions in aircraft control (with no failures), while the MMAE method was developed mainly to handle failures. The main difference between these two methods is the manner in which the controller is designed and implemented. In the MMAC method, a separate controller is associated with each KF, and the control input is the probability-weighted average of the individual control inputs, whereas in MMAE-based control, a single controller is used [21], which can be scheduled using probability-weighted parameter estimates. The MMST method involves switching to the model that is 'closest' to the failed plant and adapting from there.

The MMAE-based adaptive control approach has been investigated extensively for aircraft control in the presence of failures and has shown considerable promise. In [25], the MMAE method for FDI was extended to include actuator failures with constant unknown biases. This method, called the Extended MMAE (EMMAE) method, was also applied to nonlinear systems by using banks of Extended Kalman Filters (EKF), resulting in a promising method for actuator FDI under varying flight conditions.

The MMAE and EMMAE-based FDI methods are based on hypothesis testing using parallel banks of KFs or EKFs, which have a high complexity and require considerable real-time computation. It was reported in [22] that the MMAE method showed promise but indicated some difficulty in the case of multiple failures and simultaneous actuator and sensor failures. The results in [25], which considered only actuator faults, indicated a substantial time delay (several seconds) in identifying the faults, and proposed a supervisory module that generates an auxiliary excitation signal which can improve the speed of FDI. The resulting FDI scheme was able to detect and isolate actuator faults in 2-4 sec.

In safety critical applications such as aerospace vehicles, it is often highly desirable- if not essential- to detect and identify faults very quickly (less than a second) in order to maintain stability, maneuverability, and safe operation. This may require a substantial reduction of complexity, which is also desirable from the point of view of implementation, validation, and verification that will be required for certification. Another issue in this regard is that analytical results (such as closed-loop stability using the MMAE state estimate for feedback) need to be addressed. Furthermore, the results of the MMAE-based methods (i.e., conditional probabilities used to identify faults) depend on the values of the process- and measurement- noise. While the sensor noise variance is provided by the vendor or can be obtained experimentally, the process noise covariance is not known and is usually chosen by the designer. This calls into question the accuracy of the computed conditional probabilities. In addition, it is highly desirable to analytically establish conditions under which single and multiple faults (in *both* actuators and sensors) are identifiable. This paper aims to address these issues and to simplify the MM-based approach to FDI and adaptive control.

We propose an indirect adaptive control scheme for fault tolerance based on the EMMAE method wherein unknown actuator and sensor faults are rapidly detected and estimated using a bank of constant-gain Kalman filters (KFs). The system is assumed to have groups of actuators and groups of sensors, with each group having multiple similar actuators or sensors, some of which may fail in unknown positions. The proposed FDI approach aims to simplify the MM-based FDI methods by investigating the use of constant-gain KFs and simpler decision process based on residual covariances. After detection and estimation of the faults, control reconfiguration is performed to mitigate the effect of failed components and to obtain stability and satisfactory performance. The utility of the proposed approach lies in its ability to provide quick fault detection and identification as well as nearly instantaneous reconfiguration with low computational overhead, which makes feasible the use of a large number of failure models. Analytical conditions for identifiability of actuator and sensor faults are given, and simulation results are presented for an aircraft flight control problem. It is shown that actuator faults and sensor biases cannot be identified simultaneously using the current MM approach. To alleviate this problem, some alternative methods for sensor FDI using redundant sensors are suggested. The results demonstrate that the method can rapidly and accurately detect and estimate the faults and provide safe operation as well as satisfactory performance. The purpose of this paper is to focus on the FDIR problem; therefore the nominal system parameters are assumed to be known.

The paper is organized as follows. Section II addresses the actuator failure problem and proposes a simplified MM-based adaptive (FDIR) scheme. Section III presents simulation results for application to an aircraft flight control problem. Section IV considers sensor fault detection and identification, as well as the case with simultaneous actuator and sensor faults. Section V includes simulation results with sensor faults and simultaneous actuator and sensor faults. Section VI contains a discussion of the issues and Section VII presents some concluding remarks.

II. Actuator Failures

We consider systems having several (m) groups of actuators wherein each group has multiple similar actuators. The system is represented as:

$$\dot{x} = Ax + b_1 \sum_{i=1}^{v_1} u_{1i} + b_2 \sum_{i=1}^{v_2} u_{2i} + \dots + b_m \sum_{i=1}^{v_m} u_{mi} + v_0 \quad (1)$$

$$y = Cx + w \quad (2)$$

where $x \in R^{n \times 1}$, $u_{ji} \in R$, $A \in R^{n \times n}$, $b_i \in R^{n \times 1}$ ($i=1,2,\dots,m$), $C \in R^{l \times n}$ respectively denote the state vector, i th input in the j th actuator group, the system matrix, the input matrix for the i th actuator group, and the sensor output matrix; $v_0 \in R^n$, $w \in R^l$ are process noise and sensor noise (assumed to be white and Gaussian) having covariance intensities V_0 and W .

A. Actuator Failure Model

We consider the following type of actuator failures, wherein some of the actuators are stuck at some unknown constant values. Suppose m_j out of v_j actuators in the j th actuator group fail. Let Ω_j denote the set of indices corresponding to the failed actuators in the j th group

A failure is modeled as

$$u_{ji} = \bar{u}_{ji}, i \in \Omega_j, \text{ for } t \geq t_{ji}, j = 1, 2, \dots, m \quad (3)$$

That is, the i th actuator in the j th group gets stuck at a constant value \bar{u}_{ji} at time t_{ji} . The failures are assumed to occur at unknown instants of time t_{ji} .

Assuming that at least one actuator within each group is working, Eq. (1) can then be written as

$$\dot{x} = Ax + b_1 \sum_{i \notin \Omega_1} u_{1i} + \dots + b_m \sum_{i \notin \Omega_m} u_{mi} + B\bar{U} = Ax + B \begin{bmatrix} \sum_{i \notin \Omega_1} u_{1i} \\ \vdots \\ \sum_{i \notin \Omega_m} u_{mi} \end{bmatrix} + B\bar{U} + v_0 \quad (4)$$

where

$$B = [b_1 \ b_2 \ \dots \ b_m] \quad (5)$$

$$\bar{U} = [\bar{U}_1 \ \bar{U}_2 \ \dots \ \bar{U}_m]^T; \ \bar{U}_j = \sum_{i \in \Omega_j} \bar{u}_{ji} \quad (6)$$

Remark- In (4), we had assumed that at least one actuator within each group is working. However, if some of the groups experience failures of all actuators within the group, (4) is modified as

$$\dot{x} = Ax + BF^a \begin{bmatrix} \sum_{i \notin \Omega_1} u_{1i} \\ \vdots \\ \sum_{i \notin \Omega_m} u_{mi} \end{bmatrix} + B\bar{U} + v_0$$

where F^a denotes an $m \times m$ diagonal matrix with unit entries corresponding to groups that have at least one functioning actuator, and zero entries corresponding to groups in which all actuators have failed. (If at least one actuator in each group is working, $F^a = I$).

In this approach, we do not try to determine *which* actuator within a group has failed, but rather *how many* actuators have failed in each group, as well as the failure value \bar{U}_j . The number of failure states is $n_f = \prod_{j=1}^m (v_j + 1)$

(including the “no failure” case), which can be a large number, especially if multiple failures are possible. For example, an aircraft having doubly redundant elevator, ailerons, and rudder will have 27 actuator failure states.

B. The Adaptive Control Scheme

The objectives of the adaptive control scheme are to

- i) Detect the actuator failure
- ii) Identify the failure: determine
 - a. Which actuator groups have failed actuators
 - b. How many actuators within each group have failed, and
 - c. Failure value estimates
- iii) Reconfigure the controller using the non-failed actuators to
 - a. Mitigate/cancel out the effect of the fault ($B\bar{U}$) to the maximum extent possible
 - b. Redesign controller gains for satisfactory performance

The first task is to determine the conditions under which the actuator failures will be identifiable from the sensor measurements.

C. Identifiability of actuator failures

The approach to FDI considered in this paper is based on the MM approach which uses a bank of Kalman-Bucy filters (KBF), each of which is tuned to a different failure-state model. The model that is the most likely representation of the actual situation (indicated by hypothesis testing based on criteria such as covariance of the residuals) identifies the failure state. An advantage of this approach is that each failure state model is still linear; therefore KBF (rather than EKF) can be used for fault detection and estimation. Following the approach of [25], since the actuator faults are assumed to be constant, they can be estimated by augmenting the unknown \bar{U} to the system dynamics and designing a KBF. The augmented system is:

$$\dot{\xi} := \begin{bmatrix} \dot{x} \\ \dot{\bar{U}} \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} \xi + \begin{bmatrix} B \\ 0 \end{bmatrix} \begin{bmatrix} \sum_{i \in \Omega_1} u_{1i} \\ \vdots \\ \sum_{i \in \Omega_m} u_{mi} \\ 0_{m \times 1} \end{bmatrix} + v \quad (7)$$

$$y = [C \ 0_{l \times m}] \xi + w \quad (8)$$

where $v = [v_0^T, v_U^T]^T$, v_U being a fictitious noise added for estimating \bar{U} . The following theorem gives a necessary and sufficient condition for fault identifiability. It is assumed that B and C have a full column- and row-ranks.

Theorem 1. For the system described in (4), the actuator faults (\bar{U}) in (6) are identifiable if and only if (C, A) is observable, $l \geq m$, and the system (C, A, B) has no transmission zeros at the origin.

Proof- Denote

$$A_\xi = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}, \quad C_\xi = [C \ 0_{l \times m}] \quad (9)$$

The fault is identifiable if and only if $[C_\xi, A_\xi]$ is observable. Applying the PBH rank test, this pair is observable if and only if

$$\rho \begin{bmatrix} sI - A & -B \\ 0 & sI_m \\ C & 0 \end{bmatrix} = n + m \text{ for } s = \lambda(A), \text{ and } s = 0 \quad (10)$$

where $\rho[\cdot]$ denotes the matrix rank. Since (C, A) is observable, the first n columns of the test matrix in (10) are linearly independent for all s . For $s \neq 0$, the last m columns form an independent set and are also independent of the first n columns, i.e., $\rho = n + m$. At $s = 0$, $\rho = \min(n + m, n + l)$ if and only if the system (C, A, B) has no transmission zeros at the origin. Thus the condition (10) can hold if and only if $l \geq m$ and (C, A, B) has no transmission zeros at the origin. ■

Remark 1- If $l > m$, (C, A, B) is non-square system and almost always has no transmission zeros; thus the faults are identifiable. Also, if $l = n$ and C is nonsingular, it can be readily shown that the condition (10) holds and the faults are identifiable.

Remark 2- The condition in Theorem 1 corresponds to the models for which at least one actuator in each group has failed. For cases where some actuator groups have no failures, the “ B ” matrix in (9) would consist of only columns corresponding to actuator groups having at least one failure. Thus there are $2^m - 1$ values of the “ B ” matrix in (with varying column dimension) corresponding to all the models when there is at least one failure.

D. Failure Detection/Estimation Filters

A constant-gain KBF is designed for each of the n_f failure models:

$$\dot{\hat{\xi}} = A_\xi \hat{\xi} + \begin{bmatrix} B \\ 0 \end{bmatrix} \begin{bmatrix} \sum_{i \in \Omega_1} u_{1i} \\ \vdots \\ \sum_{i \in \Omega_m} u_{mi} \\ 0_{m \times 1} \end{bmatrix} + L(y - C\hat{x}) \quad (10)$$

where L is the Kalman gain, which is time-varying in general. Because the system is linear and time-invariant (between failures), L can quickly converge to a steady-state value; therefore we shall investigate the use of constant-gain KBFs as FDI filters. This permits the use of pre-designed KBFs, resulting in significant reduction in real-time computation, which in turn would make it possible to handle a large number of models (failure states). Note that a separate KBF is designed for each actuator state described by the set $(\Omega_1, \Omega_2, \dots, \Omega_m)$.

E. Discrete-Time Implementation

From the implementation point of view, the discrete-time formulation is more suitable than continuous time since the Kalman filters[†] (KFs) can be easily implemented in discrete-time. Thus (7), (8) are replaced by

$$\xi(k+1) := \begin{bmatrix} x(k+1) \\ \bar{U}(k+1) \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \xi(k) + \begin{bmatrix} B \\ 0 \end{bmatrix} \begin{bmatrix} \sum_{i \in \Omega_1} u_{1i}(k) \\ \vdots \\ \sum_{i \in \Omega_m} u_{mi}(k) \\ 0_{m \times 1} \end{bmatrix} + v(k) \quad (11)$$

$$y(k) = [C \ 0_{l \times m}] \xi(k) + w(k) \quad (12)$$

where, without changing the notation, A, B denote the *discretized* system- and input-matrices. The statement of Theorem 1 changes to the following:

[†] *Kalman filter* refers to the original discrete-time filter; the *Kalman-Bucy filter* refers to the continuous-time version

Theorem 1a. For the system described in (11), (12), the actuator faults are identifiable if and only if (C, A) is observable, $l \geq m$, and the system (C, A, B) has no transmission zeros at $z=1$. ■

The Kalman filter (one for each actuator failure state) is given by

$$\hat{\xi}(k) = \hat{\xi}(k/k-1) + L(k)[y(k) - C\hat{x}(k/k-1)] \quad (13)$$

where

$$\hat{\xi}(k/k-1) = A_{\xi}\hat{\xi}(k-1) + \begin{bmatrix} B \\ 0 \end{bmatrix} \begin{bmatrix} \sum_{i \in \Omega_1} u_{1i}(k-1) \\ \vdots \\ \sum_{i \in \Omega_m} u_{mi}(k-1) \\ 0_{m \times 1} \end{bmatrix} \quad (14)$$

The Kalman gain update equations may be found in standard texts. (In this paper, the use of constant-gain KFs is investigated in order to reduce the complexity and real-time computational requirements).

F. Failure Identification Criteria

In the standard multiple model hypothesis testing approach [20, 21], to detect and identify the failure, a KF is designed for each of the n_f actuator state models. To determine which model correctly represents the actual actuator failure state, the conditional probability for the i th model is computed as

$$p_i(k) = \frac{p(\theta_i / Y_k) = p(y_k / \theta_i, Y_{k-1}) p_i(k-1)}{\sum_{j=1}^{n_f} p(y_k / \theta_j, Y_{k-1}) p_j(k-1)} \quad (15a)$$

where

$$p(y_k / \theta_i, Y_{k-1}) = \frac{e^{-\frac{1}{2} r_i^T(k) R_i^{-1}(k) r_i(k)}}{(2\pi)^{l/2} |R_i(k)|^{l/2}}, \quad i = 1, \dots, n_f \quad (15b)$$

θ_i represents the i th model, $Y_k = \{y(0), y(1), \dots, y(k)\}$, $\hat{x}_i(k)$ is the state estimate based on the i th model and $r_i(k) = y(k) - C\hat{x}_i(k/k-1)$ is the residual corresponding the i th model. The model having the largest $p_i(k)$ is declared to be the correct model, and the state estimate is given by a weighted sum of the individual KF state estimates weighted by the corresponding conditional probabilities:

$$\hat{x}(k) = \sum_i \hat{x}_i(k) p_i(k) \quad (16)$$

An alternate method would be to just use the state estimate $\hat{x}_i(k)$ corresponding to the model with the highest $p_i(k)$.

This approach has a high requirement for real time computation. In addition, since the process noise covariance not known but is assumed in the KF design, the calculated probabilities used in decision-making do not necessarily represent the true probabilities. Therefore we propose a simpler heuristic method, i.e., to compute and compare the residual norm squared, $Tr[R_i(k)] = E[r_i^T(k) r_i(k)]$, which can be also be weighted by $R_i^{-1}(k)$ or $W^{-1}(k)$, and choose the model (and the state estimate) that corresponds to the smallest value.

G. Reduced Actuator Effectiveness

Another type of actuator anomaly is reduction of actuator effectiveness {actuator “fading”} by an unknown factor $\gamma \in [0, 1]$. That is,

$$u_{actual} = \gamma u_{desired}$$

It is possible to address such an anomaly in the multiple model, multiple actuator framework by using quantized levels of γ , the actuator “fading factor”. For example, with 11 levels of actuator effectiveness, $\gamma = \{0, 0.1, 0.2, \dots, 1\}$, this would be similar to having 10 identical actuators in the group, some of which may fail and produce zero actuation. Because there is no bias to be estimated, state equation augmentation as in (7) is not needed.

H. Controller Reconfiguration

After the failure state (model) is determined, it will be necessary to reconfigure the controller. Controllers tuned for each models can be pre-designed using a chosen method, such as LQG, H_2 , or H_∞ - based controller, provided that (A, BF^a) is controllable. In addition, the control input must also cancel the effect of the fault, represented by the term $B\bar{U}$. If there are one or more non-failed actuators within each group, the corresponding component $b_j\bar{U}_j \approx b_j\hat{\bar{U}}_j$ can be cancelled (provided that sufficient actuation authority is available), since the failure state (and therefore the number of failed actuators within each group) has been determined. If all actuators within a group have failed, complete cancellation would be possible only if the columns of B corresponding to non-failed actuators span the space of columns corresponding to the failed ones. If cancellation of the bias term is not possible, its effect can be minimized using a least-squares approach.

The FDIR scheme considering two elevators is illustrated in the Figure 1.

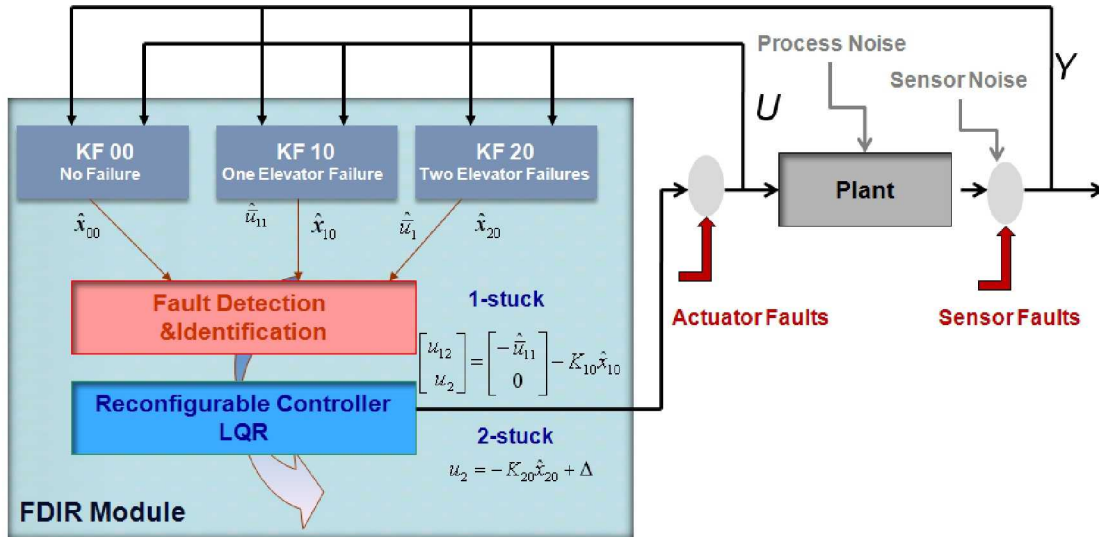


Figure 1. Indirect Adaptive Control Scheme with FDIR

This procedure is demonstrated by an example application to aircraft flight control.

III. Example Application- Aircraft Flight Control with Actuator Failures

In this section, we present simulation results to demonstrate the performance of the proposed indirect adaptive control scheme in the presence of actuator failures, which include stuck actuators as well as reduced throttle effectiveness.

A. Problem description

For the purpose of testing the method and the decision criteria described in Section II, we consider a fourth-order longitudinal dynamics model of a large transport aircraft in a cruise condition. The state variables are: pitch rate q (deg/sec), true airspeed v (m/sec), angle of attack α (deg), and pitch angle θ (deg). The main actuator consists of two elevators (each elevator consists of a pair of control surfaces, one on each side, that operate symmetrically). The two elevator inputs are u_{11} and u_{12} . The total engine thrust (equal in all engines) is used as a secondary actuator.

The individual engine thrusts have been aggregated to produce a single control input u_2 . The nominal model (ignoring noise) is

$$\begin{aligned}\dot{x} &= Ax + b_1(u_{11} + u_{12}) + b_2u_2 \\ y &= Cx\end{aligned}\quad (17)$$

where $x = [q \ v \ \alpha \ \theta]^T$;

The system matrices (continuous-time) are

$$A = \begin{bmatrix} -0.6803 & 0.0115 & -1.0490 & 0 \\ -0.0026 & -0.0062 & -0.0815 & -0.1709 \\ 1.0050 & -0.0344 & -0.5717 & 0 \\ 1.0000 & 0 & 0 & 0 \end{bmatrix}, \quad b_1 = \begin{bmatrix} -44.5192 \\ 0 \\ -11.4027 \\ 0 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 0.8824 \\ 1.3287 \\ -0.0401 \\ 0 \end{bmatrix}, \quad C = I_4 \quad (18)$$

We assume that fault-free (although noisy) measurements of the states are available, i.e., there are no sensor faults and only actuator faults are considered. The two groups of independent actuators are elevator and throttle. We define a failure of each elevator as in Eq. (3), i.e., stuck in an unknown position. For the purpose of this example, the throttle is treated as a single input that is subject to a fading factor (i.e., reduced effectiveness factor) γ_T of: 1 (full effectiveness), 0.75, 0.50, or 0.25. (This would be equivalent to having four equal thrust actuators, of which up to 3 actuators may fail and produce zero thrust ($\bar{u}_{2i} = 0$)). The corresponding failure states are shown in Table I.

Table 1: Actuator failure models

Elev \ Thrust	0: $\gamma_T = 1$	1: $\gamma_T = 0.75$	2: $\gamma_T = 0.50$	3: $\gamma_T = 0.25$
0: No failure	$Model_{00}$	$Model_{01}$	$Model_{02}$	$Model_{03}$
1: One failure	$Model_{10}$	$Model_{11}$	$Model_{12}$	$Model_{13}$
2: Two failures	$Model_{20}$	$Model_{21}$	$Model_{22}$	$Model_{23}$

The system is discretized at a sampling rate of 100Hz. A bank of 12 KFs will be needed for performing FDI.

We shall first consider the case when there is no engine failure, i.e., there are three failure states corresponding to the first column of Table I. We will need to construct 3 KFs corresponding to models (0,0), (1,0) and (2,0) as described in Section II. In designing the KFs, we choose process noise covariance intensity V that provides a KF design with good dynamic characteristics (e.g., sufficiently rapid error decay), while the sensor noise covariance intensity W is determined by the instrumentation. The noise covariance intensities (continuous-time) used in this example are

$$V_0 = \text{diag}(0.1160, 2.3094, 0.9238, 0.7167); \quad W = \text{diag}(0.04565, 0.048129, 0.0567, 0.019440) \quad (19)$$

The equivalent discrete-time covariance intensities are approximated by dividing the continuous-time intensities by T , the sampling period.

When the state vector is augmented with \bar{U} , the process noise covariance intensity is appropriately augmented with additional diagonal terms corresponding to variance intensities of the fictitious noise v_U . The three KFs are based on discrete-time versions of the following models:

1) $Model_{00}$: no failures

$$\begin{aligned}\dot{x} &= Ax + b_1(u_{11} + u_{12}) + b_2u_2 \\ y &= Cx\end{aligned}\quad (20)$$

2) $Model_{10}$: one elevator is stuck

In this case, one elevator (e.g., u_{11}) is stuck at a position \bar{u}_{11} . The state vector is augmented with \bar{u}_{11} :

$$\begin{bmatrix} \dot{x} \\ \dot{\bar{u}}_{11} \end{bmatrix} = \begin{bmatrix} A & b_1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \bar{u}_{11} \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \end{bmatrix} u_{12} + \begin{bmatrix} b_2 \\ 0 \end{bmatrix} u_2 \quad (21)$$

3) $Model_{20}$: both the elevators are stuck.

$$\begin{bmatrix} \dot{x} \\ \dot{\bar{u}}_1 \end{bmatrix} = \begin{bmatrix} A & b_1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \bar{u}_1 \end{bmatrix} + \begin{bmatrix} b_2 \\ 0 \end{bmatrix} u_2 \quad (22)$$

Where:

$$\bar{u}_1 = \bar{u}_{11} + \bar{u}_{12} \quad (23)$$

B. Elevator Failure Detection and Identification Results

It is assumed that the aircraft is in a cruise condition, with a nominal LQ state feedback control law. Suppose one elevator fails at $t = 5$ sec and gets stuck at 4 deg. (the angle is relative to the trim value). Then at $t = 15$ s, the second elevator also fails and is stuck at -1 deg. (Since the objective is to detect and identify faults, the control law is not reconfigured in these initial simulations). We first present the fault detection results based on the EMMAE method [25]. Figure 2 shows the probability indices (i.e., the conditional probabilities defined in Eq 15) for the three models. In the simulations performed, the MMAE method was not able to accurately detect the first elevator fault at $t = 5$ s, as indicated by the probability index of $Model_{00}$ (no faults) which remains at around 1.0 and both of the indices of $Model_{10}$ (one faulty elevator) and $Model_{20}$ (two faulty elevators) stay close to 0 for $0 < t < 15.5$ s. Also, it can be seen from Figure 2 that the probability index of $Model_{10}$ increases to 1.0 at $t = 15.5$ s and then slowly decreases to 0.2 (the threshold) at $t = 37$ s, while, the probability index of $Model_{20}$ increases to 0.8 (threshold). This suggests that although an elevator fault is detected, it is inaccurately identified as single elevator fault during $15.5 \text{ s} < t < 37 \text{ s}$ while the actual fault in both elevators occurred at 15 sec.

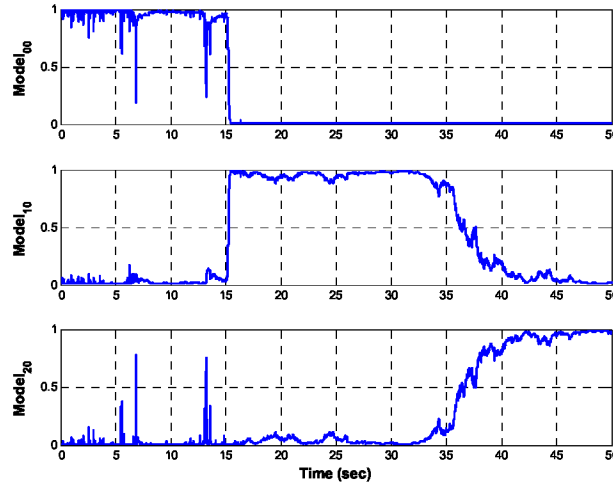


Figure 2. Detection results for the MMAE method.

We shall next present simulation results for fault detection using constant-gain KFs and unweighted and weighted residual variances of the three KFs. The inverse of the residual covariance matrix was used as the weight in the weighted residual variance method. The decision criterion is to choose the model with the smallest residual

variance. The results for the unweighted and weighted residual variances are shown in Figures 3 and 4. The top two plots in Figure 3A, 3B show the time-histories of the two elevators. As stated previously, the nominal state feedback control law is assumed to be on (with no reconfiguration) throughout the simulation. It can be seen that, even without reconfiguration, Elevator 2 tries to compensate for the failure of Elevator 1 at $t=5s$. The three lower plots in Figure 3A shows the unweighted residual variances for the three models, while the weighted residuals are shown in Figure 3B. The actual and estimated elevator failure values are shown in Figure 5. It can be seen that both the failure state and the failure value are correctly identified within about 0.5s. However, after the occurrence of the first fault at $t=5s$, the difference between the residual variances of $Model_{10}$ and $Model_{20}$, although visible, is not significant, (Figures 4A and 4B), which implies that the residual-based FDI method may not always be able to clearly distinguish between the one-elevator fault and the two-elevator fault. However, the two-elevator fault, which occurs at $t=15s$, is clearly distinguished from the one-elevator fault. The un-weighted and weighted residual variance methods gave a similar performance. Several failure cases and scenarios were simulated similar to this one, and gave similar results. In all cases, the failure value was identified rapidly and accurately.

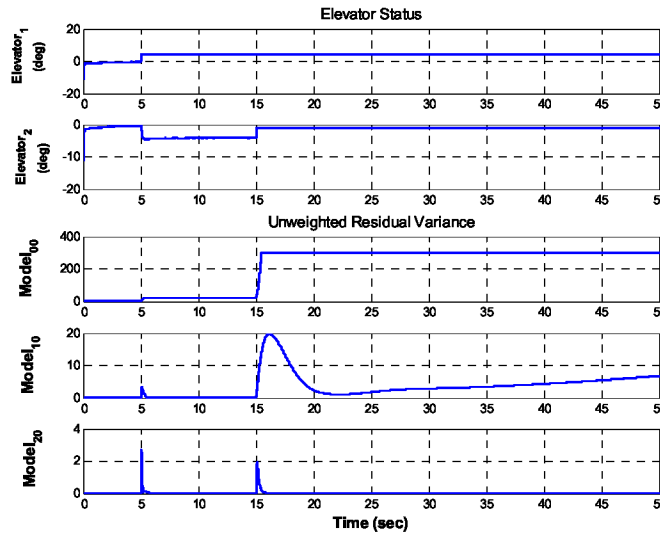


Figure 3A: Unweighted residual signals of three models for elevator fault detection

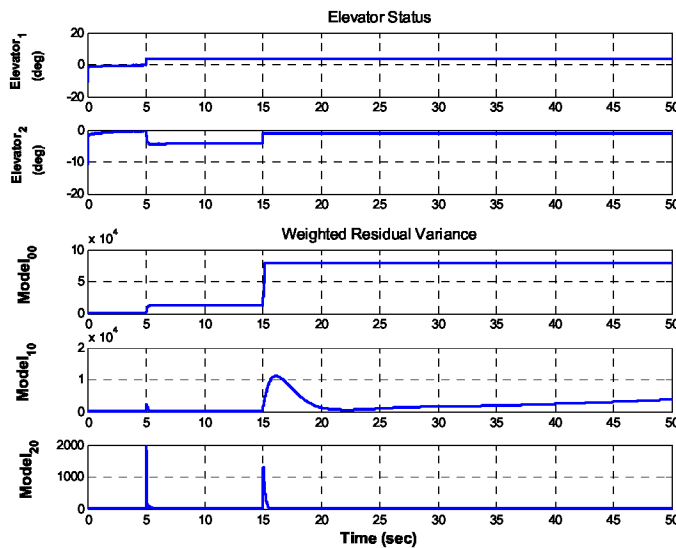


Figure 3B: Weighted residual signals of three models for elevator fault detection

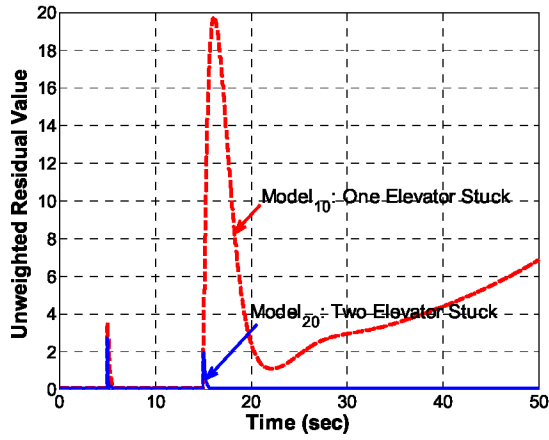


Figure 4A. Comparison of unweighted residual signals: Model 1 VS Model 2

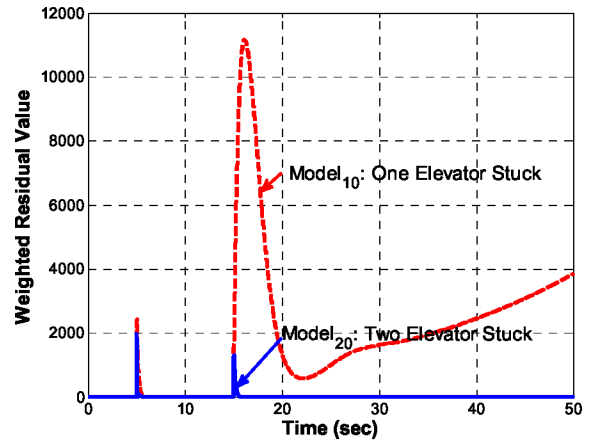


Figure 4B. Comparison of weighted residual signals: Model 1 VS Model 2

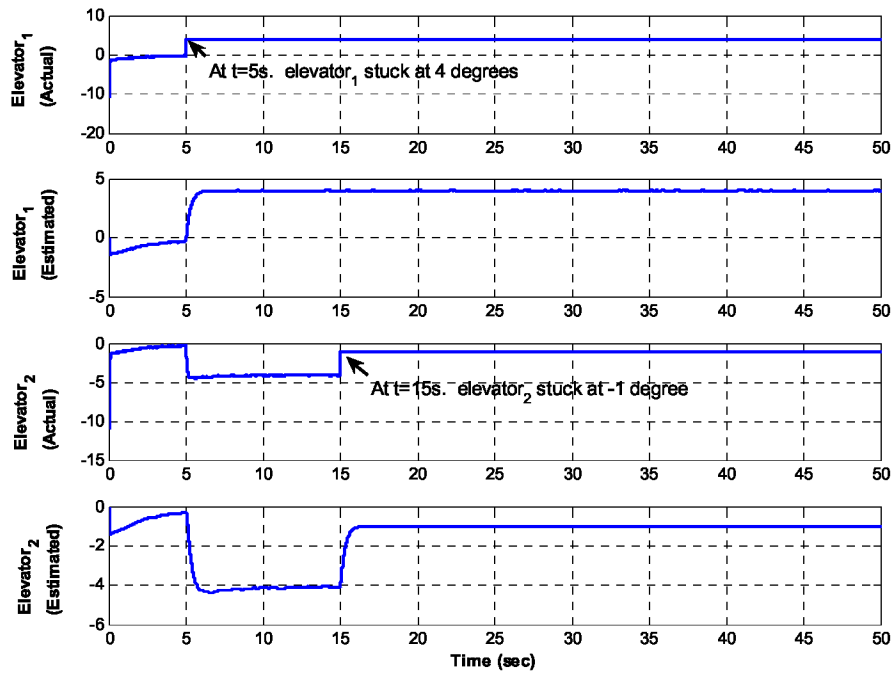


Figure 5. Estimation of the elevator status.

C. Throttle Fading Failure Identification Results

The case with no control surface failures but with throttle fading is considered next using four models: $Model_{00}$ ($\gamma_T = 1$, no fading), $Model_{01}$ ($\gamma_T = 0.75$, 25% fading), $Model_{02}$ ($\gamma_T = 0.5$, 50% fading), and $Model_{03}$ ($\gamma_T = 0.25$, 75% fading). Suppose throttle fading occurs as shown in Figure 6, that is, during $0 \leq t < 0.5s$, 0% fading ($\gamma_{actual} = 1$); during $0.5s < t < 5s$, 80% fading ($\gamma_{actual} = 0.2$); during $5s < t < 10s$, 5% fading ($\gamma_{actual} = 0.95$); during $10s < t < 15s$, 30% fading ($\gamma_{actual} = 0.7$); for $t > 15s$, 45% fading ($\gamma_{actual} = 0.55$). The actual fading factors are deliberately chosen to be slightly different from those in the models. As in the previous simulations, the nominal state feedback control

law was on throughout the simulation. Figure 7 shows the ideal (no fading) and actual thrust. Using the MMAE method, as shown in Figure 8A, only the fading at 0.5s could be identified. With the residual variance method, as shown in Figure 8B, the model with the smallest residual variance accurately suggests that relative throttle fading occurred at 0.5s, 5s, 10s and 15s, respectively. Figure 9, 10, 11, 12 show the comparisons of the residual signals of the four models around 5s, 10s and 15s, where the differences in the residual variances are magnified for clarity. Although the fault identification is technically correct, the differences are rather small. Therefore it was concluded that the throttle fading fault detection and estimation is not satisfactory and needs further investigation. In [25], an auxiliary sinusoidal persistent excitation input was proposed with the EMMAE method for improving the speed of fault detection and identification. We are investigating similar methods for use with the residual variance method.

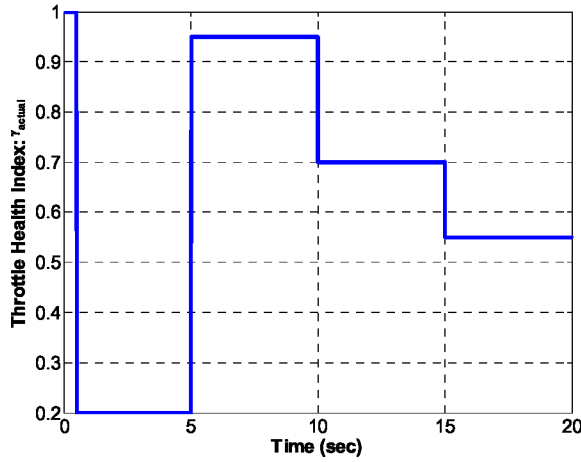


Figure 6. Throttle Health Index

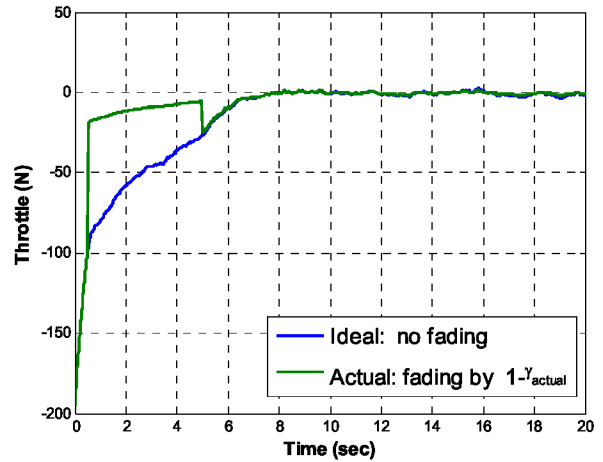


Figure 7. Ideal and actual thrust

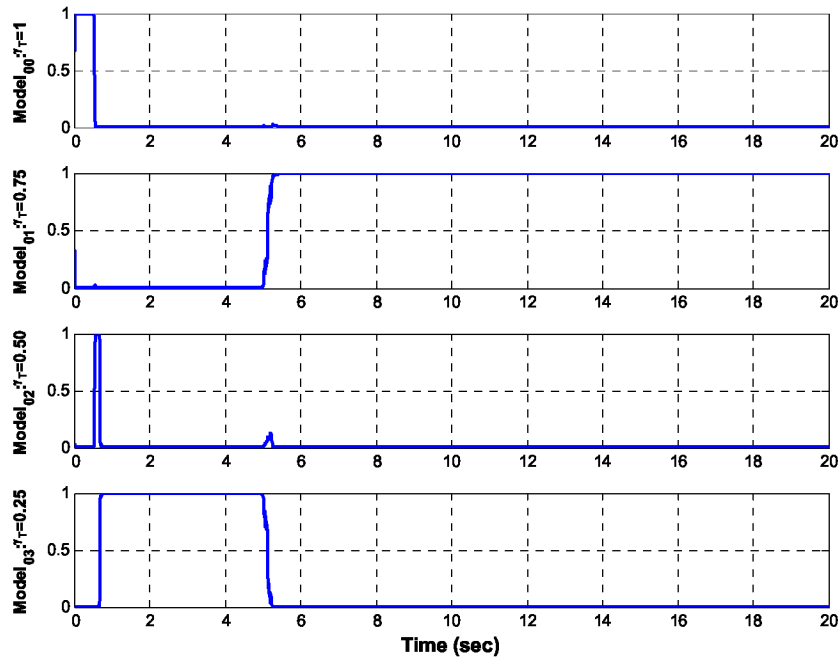


Figure 8A. Probability indices of four models for throttle fault detection (MMAE method)

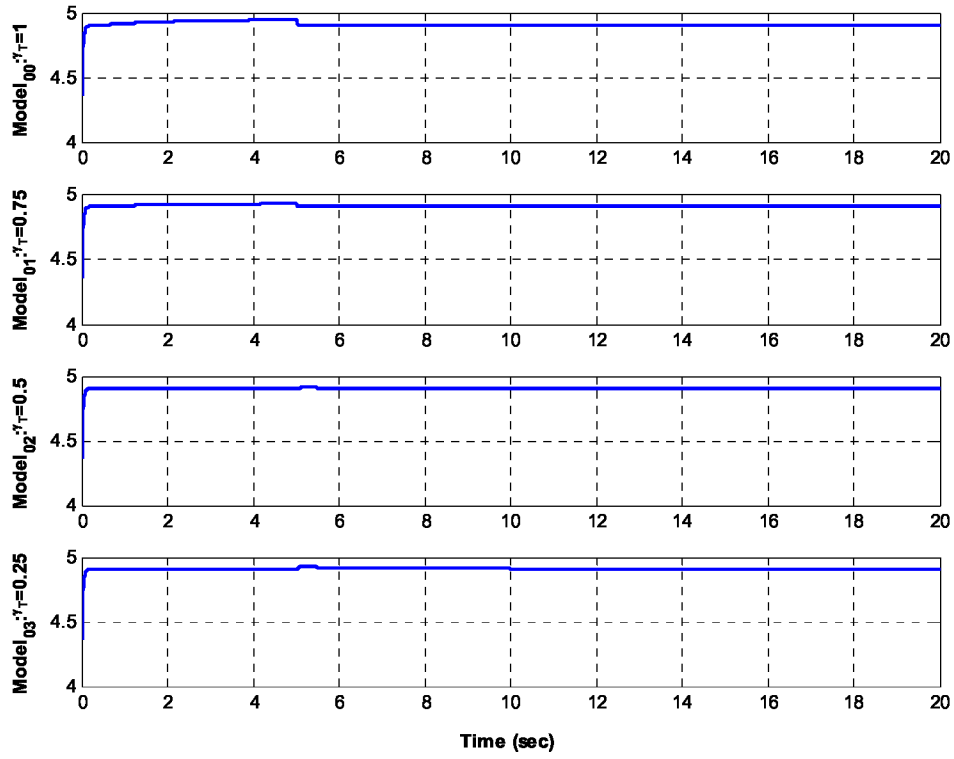


Figure 8B. Unweighted residual signals of four models for throttle fault detection

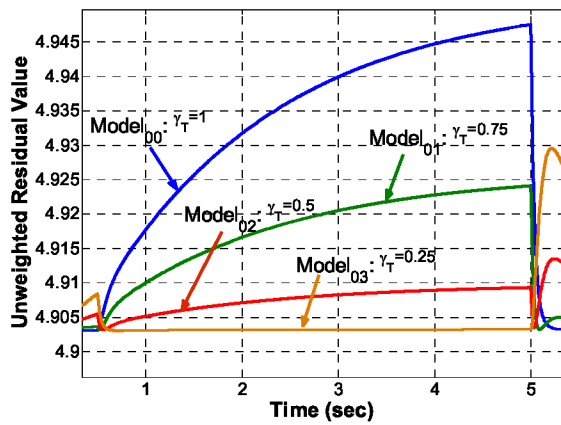


Figure 9. Comparison of residual values
 $\gamma_{actual} = 0.2$ for $0.5s < t < 5s$

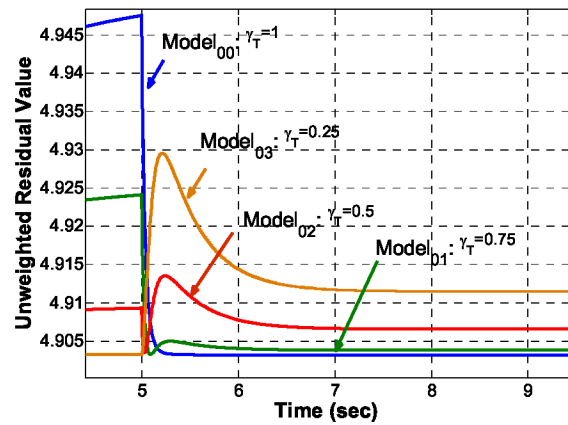


Figure 10. Comparison of residual values
 $\gamma_{actual} = 0.95$ for $5s < t < 10s$

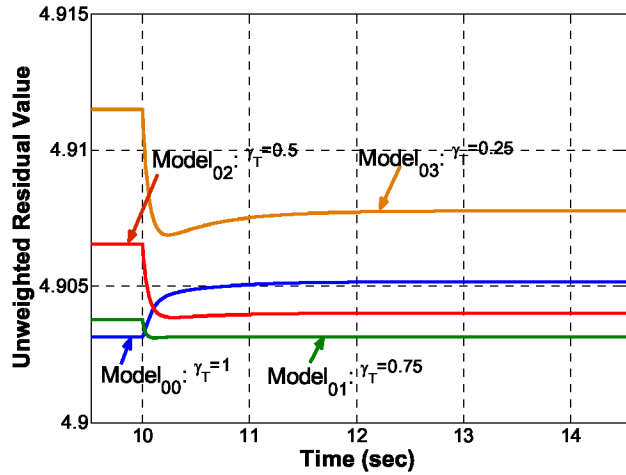


Figure 11. Comparison of residual values
 $\gamma_{actual} = 0.7$ for $10s < t < 15s$

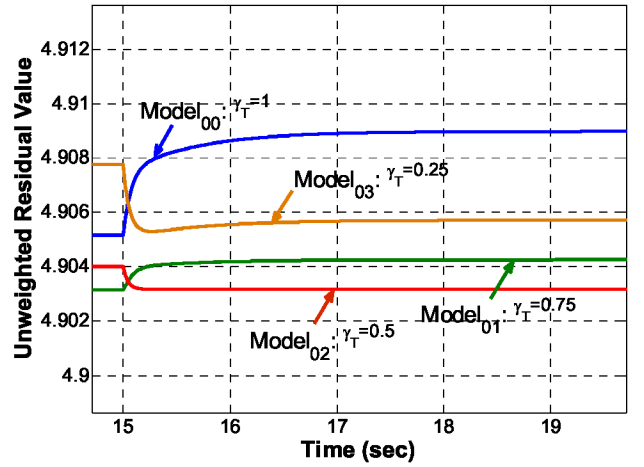


Figure 12. Comparison of residual values
 $\gamma_{actual} = 0.55$ for $t > 15s$

D. Controller Reconfiguration

The FDI results shown above used the nominal state feedback control law in all the simulations, since the purpose was to investigate the efficacy of the FDI approaches. To obtain better performance, the controller should be reconfigured after each fault is identified.

First considering the elevator faults, when one elevator failure is identified and its failure value is estimated, the command to both elevators should be:

$$u_{c2} = -\hat{u}_{11} - 2K\hat{x}_1$$

where \hat{x}_1 is the state estimate from Model 1. (In general, if there are ν_j actuators in Group j , the actuator command, consisting of the sum of the negative of the estimated bias and the feedback input, would be distributed over the estimated number of non-failed actuators). When both elevators fail at $t=15s$, the throttle is the only available control input. Although the system is still controllable, it is no longer possible to eliminate the bias since b_2 is not parallel to b_j ; therefore, the bias is minimized in the least-squares sense and the controller (LQ state feedback) is redesigned using the throttle as the only input. Figure 13A shows the response plots with elevator failures, where the throttle was reconfigured 0.5s after the elevator faults were identified, i.e. at approximately $t=5.5s$ and $15.5s$, respectively. Figure 13B shows the throttle without reconfiguration for comparison. Also, the corresponding state variables are shown in Figure 14. It is noted that the reconfiguration successfully stabilizes the system with elevator failures. However, because the bias due to faulty elevators cannot be cancelled, some of the states of the closed-loop system are not bounded. (The magnitudes of the variables in Figure 14 are based on the linear model and are not realistic; they are shown only to demonstrate the effect of non-cancellable bias).

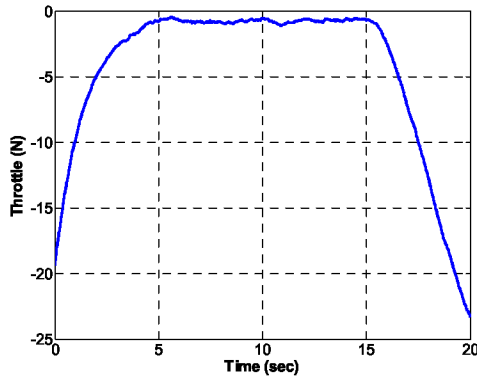


Figure 13A. Control signal with reconfiguration

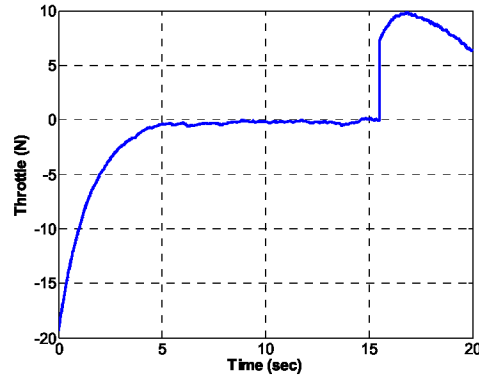
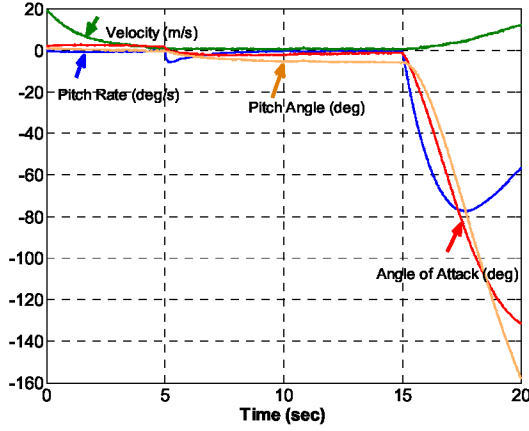


Figure 13B. Control signal without reconfiguration



Units: Angle of attack (deg), pitch angle (deg); pitch rate (deg/s); velocity (m/s)

Figure 14A. State variables without reconfiguration

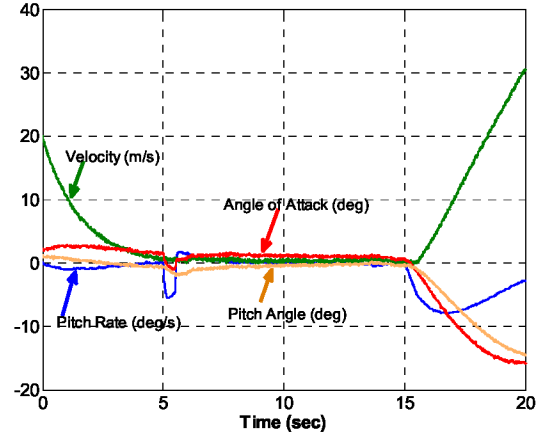


Figure 14B. State variables with reconfiguration

Although this example used LQG-based reconfigurable controller designs, methods such as H_2 and H_∞ control can be used.

IV. Sensor Fault Detection and Identification

This section addresses the sensor failure detection and estimation problem, which is the dual of the actuator failure problem addressed in Section II.

We shall first consider the case where unknown constant bias terms exist in the sensors.

A. Sensor Bias

Consider the system

$$\begin{aligned}\dot{x} &= Ax + Bu + v_0 \\ y &= Cx + \bar{y} + w\end{aligned}\tag{24}$$

where C has a full rank (i.e., no redundant sensors), $\bar{y} \in R^l$ denotes an unknown bias. As in the case of stuck-actuator failures the sensor bias can be estimated by augmenting the unknown bias term \bar{y} to the system dynamics and designing a KBF. The augmented system is:

$$\dot{\eta} = \begin{bmatrix} \dot{x} \\ \dot{\bar{y}} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \eta + \begin{bmatrix} B \\ 0 \end{bmatrix} u + v\tag{25}$$

$$y = [C \quad I_l] \eta + w\tag{26}$$

The following theorem gives a necessary and sufficient condition for identifiability of the sensor bias.

Theorem 2. For the system in (24), the sensor bias \bar{y} is identifiable if and only if (C, A) is observable and A has no eigenvalue at the origin.

Proof- The above system is observable if and only if

$$\rho \begin{bmatrix} sI - A & 0 \\ 0 & sI_l \\ C & I_l \end{bmatrix} = n + l \text{ for } s = \lambda(A), \text{ and } s = 0\tag{27}$$

Since (C, A) is observable, the first n columns of the test matrix in (27) are linearly independent for all s . For $s \neq 0$, the last m columns are independent of each other and of the first n columns, i.e., $\rho = n + l$. At $s=0$,

$$\rho \begin{bmatrix} -A & 0 \\ C & I_l \end{bmatrix} = \rho \begin{bmatrix} -A & 0 \\ C & I_l \end{bmatrix} \begin{bmatrix} I_n & 0 \\ -C & I_l \end{bmatrix} = \rho \begin{bmatrix} -A & 0 \\ 0 & I_l \end{bmatrix} = n+l \text{ if and only if } \lambda(A) \neq 0 \quad (28)$$

Discrete-time implementation- As in the case of actuator failures, discrete-time formulation is used in implementation. In this case the condition in Theorem 2 simply requires that the **discretized “A” matrix has no eigenvalues at $z=1$** .

An alternate method for sensor bias removal that is used in practice is to implement washout filters. For example, with a first-order washout filter, the filtered sensor output is given by

$$Y_f(s) = G_{wo}(s)Y(s) = \frac{\tau s}{\tau s + 1} Y(s) \quad (29)$$

where τ is the time constant. The drawbacks of a washout filter, however, are: i) along with the bias, it also removes the zero-frequency (DC) component of the signal, ii) it introduces a phase lag, and iii) it introduces a non-minimum-phase zero. To maintain the minimality of the system and to avoid unstable pole-zero cancellation, the plant should not have poles at the origin.

B. Sensor Outage and Bias- Multiple Redundant Sensors

We consider the case wherein multiple redundant sensors are available for each measurement. Suppose there are l groups of sensors, wherein each (i th) group consists of N_i identical sensors, i.e., the sensor output of the j th sensor within the i th group is

$$y_{ij} = c_i x + \bar{y}_{ij} + w_{ij} \quad i=1, \dots, l; j=1, \dots, N_i \quad (30)$$

where the overhead bar denotes the unknown bias term. Suppose the outputs of all sensors within each (i th) group are averaged to get the measurement Y_i :

$$Y_i = (1/N_i) \sum_{j=1}^{N_i} y_{ij}, \quad i=1, \dots, l \quad (31)$$

Suppose some of the sensors within each group experience an outage, e.g., if the k th sensor in the i th group fails, its output is

$$y_{ik} = [0]x + \bar{y}_{ik} + w_{ik} \quad (32)$$

In the bias estimation approach, the problem is to i) determine how many sensors within each group have failed, and ii) estimate the total bias in Y_i . Similar to the actuator failure case, this can be done by designing multiple KFs, each tuned to a different sensor failure model. For simplicity, consider the case with two groups of sensors wherein the first group has 2 identical sensors and the second group has 3 identical sensors. The three possible sensor failure states for Group 1 are: no outages, one outage, and two outages. Similarly there are 4 sensor failure states for Group 2. Thus the outputs of the two groups are given by

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = F^s Cx + \bar{Y} + w \quad (33)$$

where \bar{Y} is the effective bias vector and F^s is the sensor failure matrix which can take on $3 \times 4 = 12$ values:

$$F^s = \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix}, \quad \delta_1 = 1, 0.5, 0; \quad \delta_2 = 1, 0.667, 0.333, 0 \quad (34)$$

For the multiple model FDI method, the procedure is to augment the system dynamics as in (25), and to design 12 KBFs, each tuned to one of the 12 sensor models. In general, there would be $\prod_{i=1}^l (N_i + 1)$ sensor failure states including the no-failures and all-failures cases. As long as $(F^s C, A)$ is observable (i.e., (C, A) is observable and at least one sensor within each group is working) and A has no eigenvalues at the origin, it is possible to estimate the

bias and the state. As in the case of actuator failures, the model with the highest conditional probability or the smallest residual norm determines the correct failure state. However, based on the experience with actuator failures, it may not be possible to make a clear distinction between similar sensor failure states.

We shall next consider the case of simultaneous actuator and sensor failures.

C. Simultaneous sensor and actuator failures

If some of the actuators fail and if unknown sensor bias is simultaneously present, it would be necessary to identify the extent of the failures (i.e., how many actuators and sensors have failed), estimate the actuator biases and estimate and remove the sensor biases. In the multiple model approach, the number of actuator and sensor failure states (and therefore the number of models) would be $\prod_{j=1}^m (\nu_j + 1) \prod_{i=1}^l (N_i + 1)$ which can be very large. More importantly, it is necessary to first check the observability of the complete augmented system:

$$\dot{\chi} := \begin{bmatrix} \dot{x} \\ \dot{\bar{U}} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \chi + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \sum_{i \in \Omega_1} u_{1i}(k) \\ \vdots \\ \sum_{i \in \Omega_m} u_{mi}(k) \\ 0_{m \times 1} \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} \bar{U} + v'(k) \quad (35)$$

$$y = [C \quad 0 \quad I] \chi + w \quad (36)$$

From Theorem 2, this system is not observable because the matrix $\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}$ has eigenvalues at the origin. Thus, this

multiple model method, in its current form, cannot be applied to the simultaneous actuator/sensor failure case. It can be shown that using feedback of sensed output or state, or of the state estimate (to get non-zero closed-loop eigenvalues), does not change observability. Therefore other alternate methods must be used for sensor fault detection and identification, as discussed in the next subsection.

D. Alternate Methods for Sensor Failure Detection

When multiple redundant sensors are available, it is possible to determine which sensors are faulty. If sensor bias is not present in non-failed sensors, simple voting can be used to detect and isolate a faulty sensor. For example, in the case of triple redundancy, suppose y_1, y_2, y_3 denote the measurements from three sensors. The distances between the sensor measurements are:

$$d_{ij} = |y_i - y_j|; \quad i, j = 1, 2, 3$$

With n_s sensors, there will be $n_s(n_s - 1) / 2$ differences. For sensor K , if d_{Kj} ($j=1, 2, 3$) exceeds a threshold for two or more values of j , then sensor K is declared to be faulty and the measurements from the remaining sensors are averaged and used as the correct measurement. If there is quadruple redundancy, there would be 6 differences (d_{ij}). If d_{Kj} ($j=1, \dots, 4$) exceeds a threshold for 3 values of j , then sensor K is declared to be faulty. In the case of quad redundancy, two faulty sensors can be detected in this manner if their failure values (i.e., erroneous measurements) are mutually separated by a distance greater than the threshold.

Since additive white noise is assumed to be present in all the measurements, it is desirable to determine the distance as a running average of the difference between sensor measurements at N previous time steps, rather than using the instantaneous measurement value. i.e.,

$$d_{ij}(k) = \frac{1}{N} \sum_{l=k-N}^k |y_i(l) - y_j(l)|$$

This effectively reduces the measurement noise variance by a factor of N . The distance threshold for fault declaration can then be set at, for example, $3\sqrt{W/N}$ where W is the sensor noise variance intensity. The choice of N should be such that an N -step delay in fault identification is acceptable.

Other approaches for sensor FDI, (e.g., [26] which uses products of distances) should also be investigated.

V. Concluding Remarks

An indirect adaptive control scheme based on multiple model methods was proposed. Actuator failure detection and identification (FDI) for systems having groups of similar actuators was first addressed. The actuator failure types consisted of unknown actuators stuck in unknown positions, and reduced effectiveness (actuator fading). The approach uses a bank of Kalman filters based on multiple models, and subsequent control reconfiguration to mitigate the effect of biases caused by failed actuators, as well as to obtain stability and satisfactory performance using the remaining actuators. Conditions for actuator fault identifiability were presented, and methods for control reconfiguration were discussed. The scheme utilizes constant-gain Kalman filters and a residual-based decision criterion, which substantially reduces complexity and real-time computation needs. Results of application of the scheme to an aircraft flight control example indicate that actuator faults can be detected and estimated rapidly and accurately, although some difficulty was observed in distinguishing between similar faults and in estimating the actuator fading factor. Use of persistent excitation to increase the information content offers promise in this regard and will be addressed in future work. The dual problem of sensor FDI for a class of sensor failures (sensor biases and outages) was also addressed, and conditions for sensor fault identifiability were presented. It was shown that it is not possible to estimate simultaneous actuator and sensor bias faults, and therefore alternate methods for sensor FDI need to be investigated. A brief discussion of sensor FDI using multiple redundant sensors was presented. The focus of this paper was on FDI; therefore the system parameters were assumed to be known. Future research is planned for incorporating real-time system identification in conjunction with fault detection and estimation.

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